 DOI: 00.0000/X000000000000000X

Transport coefficients for solar and stellar dynamos

M. Ossendrijver[1] † **and P.J. Käpylä**[2]

[1]Netherlands Institute for the Near East (NINO), Witte Singel 25, 2311 BG Leiden, NL

[2]Kiepenheuer-Institut für Sonnenphysik (KIS), Schöneckstrasse 6, 79098 Freiburg, D

Abstract. The large-scale magnetic field of the Sun and solar-type stars is probably generated near the interface of the radiative core and the convection zone. One of the well-known difficulties of interface-type and other dynamo models is to explain the latitudinal distribution of magnetic fields as observed on the Sun. In this contribution new results of numerical MHD simulations of transport coefficients of magnetic fields in the rapid rotation regime, relevant for the base of the solar convection zone, are presented that may contribute to explaining the latitudinal distribution of magnetic fields observed on the Sun. A brief outlook on further numerical simulations of transport coefficients and their relevance for mean-field dynamo theory is given.

Keywords. Sun: activity, magnetic fields; stars: activity, magnetic fields; MHD

1. Introduction

The large-scale magnetic field of the Sun is thought to be generated near the bottom of the convection zone. Even though a comprehensive explanation of the solar cycle has yet to be found, theoretical and numerical investigations have provided detailed information about the mechanisms that may play a role in the dynamo process. Perhaps the most likely scenario is the interface $\alpha\Omega$ model originally due to Parker (1993). According to this model the strong toroidal magnetic field, fragments of which we observe in the form of bipolar pairs of sunspots, is generated in the tachocline, a region of strong radial shear below the convection zone. The poloidal magnetic field is formed from the toroidal one through an α effect based on passive advection of the magnetic field in the lower half of the convection zone. Some of the physical mechanisms that likely play a role in the interface model have been considered in more detail in Ossendrijver (2005a) and Ossendrijver (2005b).

Even though the interface model is designed to overcome some of the old problems related to storage of the magnetic field and too strong magnetic quenching of the α effect, other issues have remained unresolved (for a review of the solar dynamo and a discussion of current problems, cf. Ossendrijver 2003). One of them has to do with the most basic observed property of the large-scale solar magnetic field, namely its latitudinal distribution. It has been known for a very long time that sunspots are not found above a latitude of about 35 degrees.

Dynamo theory in its barest form suggests that the efficiency of magnetic field generation scales as $|\alpha_{\phi\phi}\nabla\Omega|$ for an $\alpha\Omega$ dynamo, where $\alpha_{\phi\phi}$ is the component of the α tensor that is responsible for regenerating the poloidal mean magnetic field, and $\nabla\Omega$ is the differential rotation. From helioseismological inversions it is known that in the Sun $\nabla\Omega$ is strongest near the poles (Schou et al. 1998). From basic principles one expects the α

† Present address: Altorientalisches Seminar, Universität Tübingen, Burgsteige 11, Schloss Hohentübingen, 72070 Tübingen; email: mathieu@kis.uni-freiburg.de

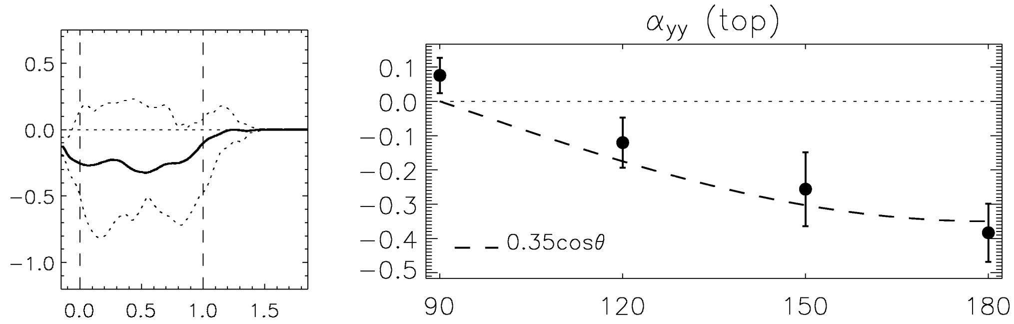

Figure 1. Results for slow rotation (Co = 1). **Left:** $\alpha_{yy} = \alpha_{\phi\phi}$ averaged over time in dimensionless units as a function of depth in the simulation box. The dimensionless unit of α is $0.01\sqrt{dg}$, that of depth is d, the thickness of the unstable region; g is the gravity acceleration. The simulation box represents a colatitude of 150 degrees (90 = equator, 180 = south pole). **Right:** $\alpha_{yy} = \alpha_{\phi\phi}$ averaged over time and over the unstable layer in dimensionless units as a function of colatitude. The dashed curve is a $\cos\theta$ fit. In the overshoot layer $\alpha_{\phi\phi}$ vanishes.

effect to vanish at the equator and peak at the poles, where the Coriolis force is strongest. Hence the dilemma is clear: the solar dynamo ought to function most efficiently at high latitudes, whereas sunspots, the main diagnostic of the deep-seated large-scale magnetic field, are observed only at low latitudes.

Several suggestions have been made in order to explain this. If the α effect results from a hydrodynamical instability in the tachocline or a buoyancy-driven instability in the magnetic storage layer, it might exhibit a maximum at low latitudes if the growth rates of the instability does so. This is not the occasion to discuss the merit of different dynamo scenarios (e.g. Ossendrijver 2003); it suffice to remark that compared to scenarios based on tachocline or buoyancy instabilities the interface model has intuitive advantages. Firstly, the 'convective' α effect is readily available in an extended region. Furthermore, since the magnetic field is just a passive agent, there is no lower limit to the field strength for it to work. In any case, there should be compelling reasons to dismiss the 'convective' α effect in favor of more exotic ones.

Previous numerical investigations of the 'convective' α effect have confirmed that it peaks at the poles (Ossendrijver et al. 2002). This might still be considered reconcilable with solar observations if magnetic flux tubes are significantly less buoyant at high latitudes than at the latitudes of sunspots. If the tubes do not rise to the surface, dynamo action near the poles remains invisible in terms of sunspots. On the other hand, recent new insights in the stability of magnetic flux tubes have made this a less likely possibility, because magnetic elements in the 'overshoot layer' are now known to be slowly but constantly heated, thereby increasing there buoyancy.

In this contribution we report on recent numerical MHD simulations of dynamo coefficients that might explain the observed distribution of large-scale magnetic fields on the Sun without invoking any enhanced stability of flux tubes at high latitudes. The motivation for the simulations has been a reassessment of the dimensionless rotation rate, the Coriolis number $\mathrm{Co} \equiv 2\Omega L/U$, which was assumed to be about unity in a previous numerical study of dynamo coefficients (Ossendrijver et al. 2002). As was in fact already reported by some authors (e.g., Küker et al. 1993), Co is considerably larger than unity near the base of the solar convection zone.

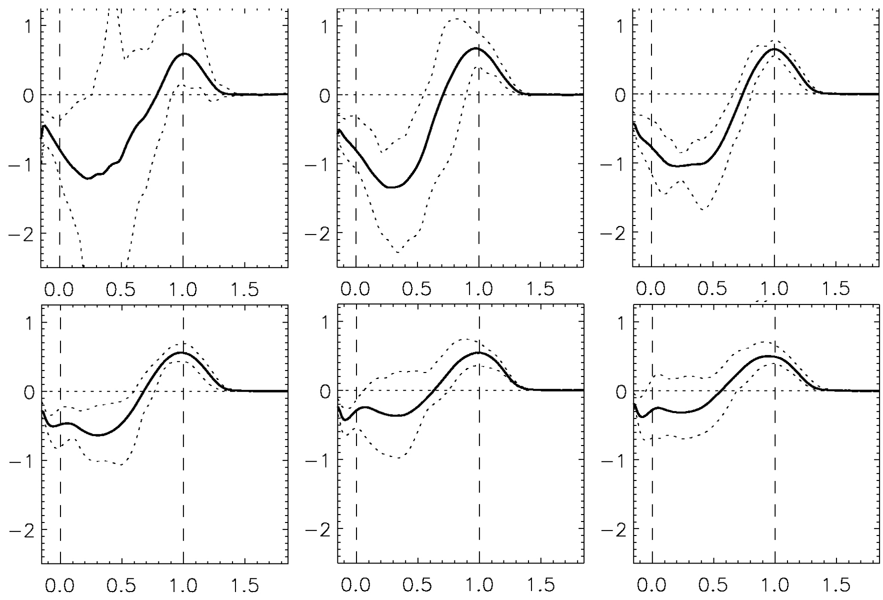

Figure 2. Results for rapid rotation (Co = 10). $\alpha_{yy} = \alpha_{\phi\phi}$ averaged over time in dimensionless units as a function of depth (z) in the simulation box. Dimensionless units are the same as in figure 1. From top left to bottom right the graphs represent simulations between 105 and 180 degrees (= south pole) of colatitude, at 15 degree intervals. Notice the rotational quenching of $\alpha_{\phi\phi}$ in the unstable region at high latitudes, and the region where $\alpha_{\phi\phi}$ has the opposite sign near $z = 1$, the base of the unstable layer.

2. Results

We have computed dynamo coefficients by performing MHD simulations of the passive advection of magnetic fields by convection in a Cartesian box representing part of the solar convection zone. A detailed description of the convection model, equations, boundary conditions, parameters, and numerical setup can be found elsewhere (Käpylä et al. 2006). The simulation domain consists of a convectively unstable layer in between two stable layers. The lower stable layer represents the 'overshoot layer' below the convection zone. As usual, the spatial resolution (128^3) is much too small to cover the wide range of length scales in the solar convection zone. Thus the values of certain dimensionless parameters such as the hydrodynamic and magnetic Reynolds numbers and Prandtl numbers are far from what they are near the base of the solar convection zone. However, the critical parameter of interest in the current investigation is the Coriolis number (or inverse Rossby number), which measures the strength of rotation, and this can be accurately reproduced.

The dynamo coefficients parametrize the assumed proportionality between the electromotive force (EMF), $\mathcal{E} \equiv \overline{\mathbf{u} \times \mathbf{b}}$, and the spatial derivatives of the mean magnetic field, $\overline{\mathbf{B}}$. All derivatives above the zeroth order one are ignored here, leading to $\mathcal{E}_i = a_{ij}\overline{B}_j$. The 9 unknown coefficients in this equation are determined by performing 3 simulations with different orthogonal imposed test fields. Mean quantities are defined as averages over the horizontal coordinates, but in order to obtain significant results a temporal average is also required. Special care is taken to ensure that the expansion up to zeroth order in the spatial derivative is a sufficient one for all the snapshots that are used in the temporal average.

The a_{ij} tensor can be expressed in terms of the α effect and the pumping vector

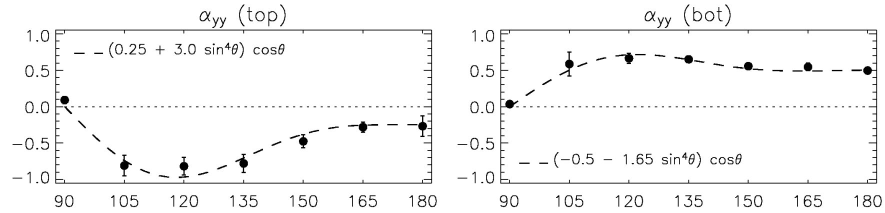

Figure 3. $\alpha_{yy} = \alpha_{\phi\phi}$ in the rapid rotation regime (Co = 10), averaged over time evaluated at two different depths in dimensionless units as a function of colatitude (90 = equator, 180 = south pole). The dashed curve is a spline fit. **Left:** $z = 0$ (unstable region). **Right:** $z = 1$ (stable layer).

according to

$$\alpha_{ij} = \frac{1}{2}(a_{ij} + a_{ji}), \tag{2.1}$$

$$\gamma_i = -\frac{1}{2}\epsilon_{ijk}a_{jk}, \tag{2.2}$$

i.e. α_{ij} and γ_i are the symmetric and antisymmetric parts of a_{ij}, respectively. The EMF can now be rewritten as $\mathcal{E} = \alpha \circ \overline{\mathbf{B}} + \gamma \times \overline{\mathbf{B}}$ and fed into the mean-field induction equation,

$$\frac{\partial \mathbf{B}_0}{\partial t} = \mathrm{curl}\left(\mathbf{U}_0 \times \mathbf{B}_0 + \mathcal{E} - \eta\, \mathrm{curl}\, \mathbf{B}_0\right). \tag{2.3}$$

Hence the pumping effect is formally equivalent to advection of the mean magnetic field. The 'true' advection term, $\mathbf{U}_0$, consists of meridional motion and differential rotation. For an $\alpha\Omega$ dynamo the most important components of the α tensor is $\alpha_{\phi\phi}$, since this is responsible for producing the poloidal magnetic field.

For slow rotation (say, Co < 5), $\alpha_{\phi\phi}$ increases approximately linearly with Co. Throughout the slow rotation regime $\alpha_{\phi\phi}$ is positive in the bulk of the convection zone on the northern hemisphere, and the dependence on latitude is approximately proportional $\cos\theta$ (Fig. 1). One qualitative change that is observed if Co approaches the fast rotation regime is that a region is formed below the convection zone proper where $\alpha_{\phi\phi}$ has the opposite sign to that in the bulk of the convection zone. For very weak rotation this region is absent. The possible role of this region for the solar dynamo is a matter of discussion.

The conclusion that the dynamo efficiency in a solar interface model ought to be highest near the poles is based on results for Co ≈ 1. However, Co ≈ 1 is not appropriate for the very base of the solar convection zone, being more representative near the top. Mixing length arguments suggest that near the bottom of the convection zone Co ≈ 10 (Käpylä 2005). As it turns out, there are significant changes in the dependence of $\alpha_{\phi\phi}$ on latitude and depth if Co increases only slightly beyond the largest value that had been considered thus far in simulations. This is illustrated in figure 2, which contains graphs of $\alpha_{\phi\phi}$ as a function of depth for different colatitudes for Co $= 10$.

Whereas in the slow rotation case there is no region near the base of the convection zone where α_{yy} has the opposite sign, such a region is now a very pronounced and robust feature found at all latitudes away from the equator. The possible relevance of this feature is discussed further below.

Furthermore, the value of α_{yy} is quenched when approaching the poles. The degree of quenching is largest in the bulk of the convection zone, but it is also observed in the opposite-sign region near $z = 1$. This might explain why the solar dynamo is most efficient not at high latitudes but at low and intermediate latitudes.

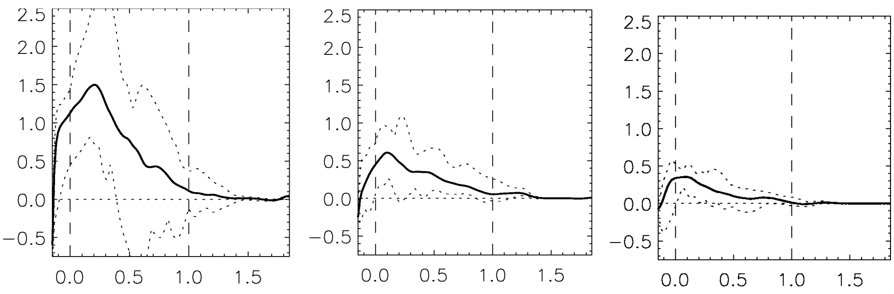

Figure 4. Results for rapid rotation (Co = 10). Latitudinal pumping effect, $\gamma_x = \gamma_\theta$, averaged over time in dimensionless units as a function of depth z in the simulation box. The graphs represent simulations at 105, 135, and and 165 degrees of colatitude. Positive γ_x signifies equatorward pumping.

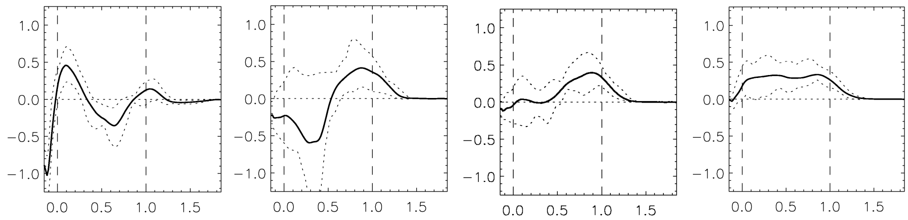

Figure 5. Results for rapid rotation (Co = 10). Radial pumping effect, $\gamma_z = \gamma_r$, averaged over time in dimensionless units as a function of depth z in the simulation box. The graphs represent simulations at 90 (= equator), 105, 120, and 135 degrees of colatitude. Positive γ_z signifies downward pumping. In the rightmost graph and for still larger colatitudes the 'normal' downward pumping known from the slow rotation case is observed throughout the domain.

In the context of the interface dynamo model it is also of interest to consider the latitudinal and radial pumping effects. Figure 4 shows the latitudinal pumping effect as a function of depth for various colatitudes. In this case the main message is that latitudinal pumping remains a robust feature of convection in the rapid rotation regime.

More surprises are observed in the behaviour of the radial pumping effect (figure 5). Whereas the radial pumping effect is always downward in the slow rotation regime (and for weak imposed magnetic fields), it now turns out to be upward at some depths in a latitudinal region close to the equator. This might have interesting consequences for the role of magnetic pumping in transporting magnetic fields into the storage layer below the convection zone, since these results suggest that close to the equator the downward pumping is inhibited near the base of the convection zone.

3. Discussion

We have presented numerical simulations of dynamo coefficients in the rapid rotation regime relevant for the base of the solar convection zone. Several interesting features are observed that do not exist in the slow rotation regime. A pronounced region of opposite-sign $\alpha_{\phi\phi}$ is formed at the base of the convection zone. The possible relevance of this feature for the interface model lies in the fact that a dynamo model based on a coefficient α_{yy} which is negative on the northern hemisphere would produce dynamo waves migrating in the correct equatorward direction also without invoking an equatorward meridional flow. Even though it is certain that there is an equatorward meridional flow at some depth in the convection zone, it has never been measured, so its speed and distribution are unknown. It is therefore uncertain whether a meridional flow would in fact have the desired effect, and one should still consider the possibility that the solar cycle is not controlled by the meridional flow, but a pure wave phenomenon.

Secondly, $\alpha_{\phi\phi}$ is rotationally quenched at high latitudes. Thirdly, the equatorward pumping effect is not quenched, but similar to what was found in the slow rotation case. The rotational quenching of $\alpha_{\phi\phi}$ at high latitudes and the equatorward pumping can aid in explaining the (apparent) absence of dynamo-generated large-scale magnetic fields at high latitudes. This might be illustrated by computing the mean magnetic field from a dynamo model that incorporates the results reported here, something that falls out of the scope of the present investigation (cf. Käpylä et al. 2006b).

Finally, the radial pumping effect exhibits a puzzling new feature in the rapid rotation regime. For a weak magnetic field the downward pumping effect was thus far considered to be a highly robust feature found at all latitudes. It now turns out that for rapid rotation radial pumping can be in the upward direction at certain depths in the convection zone, at latitudes close to the equator. No explanation is yet available for this phenomenon. How this might affect the accumulation of magnetic flux in the magnetic storage layer in the Sun and solar-type stars remains to be assessed.

Acknowledgements

M.O. acknowledges the IAU for enabling the presentation of this work by waiving the registration fee to IAU Symposium 233. P.J.K. acknowledges financial support from the graduate school for astronomy and space physics of the Finnish academy, and the Kiepenheuer Insitute for travel support. Furthermore, the Academy of Finland grant number 1112020 is acknowledged. All simulations were carried out on the BAGDAD cluster of the Kiepenheuer Institute.

References

Käpylä, P.J., Korpi, M.J., Stix, M., Tuominen, I. 2005, *A&A* 438, 403
Käpylä, P.J., Korpi, M.J., Ossendrijver, M., Stix, M. 2006, *A&A* (in press; astro-ph/0602111)
Käpylä, P.J., Korpi, M.J., Tuominen, I. 2006b, *Astron. Nachr.* (submitted)
Küker, M., Rüdiger, G., Kitchatinov, L.L. 1993, *A&A* 279, L1
Ossendrijver, M., Stix, M., Brandenburg, A. 2002, *A&A* 394, 735
Ossendrijver, M. 2003, *A&AR* 11, 287
Ossendrijver, M. 2005a, *Astron. Nachr.* 326, 166
Ossendrijver, M. 2005b, *Adv. Space Res.*, http://dx.doi.org/10.1016/j.asr.2005.07.021
Parker, E.N. 1993, *ApJ* 408, 707
Schou, J., Antia, H.M., Basu, S., et al. 1998, *ApJ* 505, 390