

# Shaping core dynamos in A-type stars: The role of dipolar fossil fields

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## ABSTRACT

*Context.* Large-scale magnetic fields of Ap/Bp stars are stable over long timescales and have typically simple dipolar geometries, leading to the idea of a fossil origin. These stars are also expected to have convective cores that can host strong dynamo action.

*Aims.* We aim to study the interaction between the magnetic fields generated by the convective core dynamo of the star, and a dipolar fossil field reminiscent of observed magnetic topologies of Ap/Bp stars.

*Methods.* We use numerical 3D star-in-a-box simulations of a  $2.2M_{\odot}$  A-type star, where the core encompasses 20% of the stellar radius. As an initial condition, we impose two purely poloidal configurations, both with a surface dipolar strength of 6 kG, and we explore different obliquity angles  $\beta$  (the angle between the magnetic and rotational axes), ranging from  $0^{\circ}$  to  $90^{\circ}$ .

*Results.* The inclusion of a poloidal field where none of the magnetic field lines are closed inside the star, does not affect the core dynamo in a significant way. Dipolar configurations where all the field lines are closed inside the star can enhance the dynamo, producing a superequipartition quasi-stationary solution, where the magnetic energy is 5 times stronger than the kinetic energy. The enhanced core dynamos have typical magnetic field strengths between 105 and 172 kG, where the strength has an inverse relation with  $\beta$ . The strong magnetic fields produce an almost rigid rotation in the radiative envelope, and change the differential rotation of the core from solar-like to anti-solar. The only cases where the imposed dipoles are unstable and decay are those with  $\beta = 90^{\circ}$ . In the rest of cases, the core dynamos are enhanced and the surface magnetic field survives keeping simple topologies like in the observations.

**Key words.** Stars: magnetic field – Stars: early-type – Magnetohydrodynamics (MHD) – Dynamo

## 1. Introduction

Large-scale magnetic fields have been observed in around 10% of main-sequence (MS) early-type stars (Moss 2001; Kochukhov & Bagnulo 2006; Landstreet et al. 2007, 2008; Grunhut et al. 2017; Shultz et al. 2019). Unlike late-type stars, whose magnetic fields have complex geometries and evolve in relatively short timescales (see e.g. Suárez Mascareño et al. 2016), the observed magnetic fields of early-type stars have simpler geometries, and they are stable over long timescales, with virtually no variability over several decades (Donati & Landstreet 2009; Briquet 2015). The observed magnetic fields from late-type stars are very likely to be due to dynamo processes operating in their convective envelopes (see e.g. Charbonneau 2020, for the solar dynamo). Numerical simulations capture many elements of these dynamos (see e.g. Käpylä et al. 2023, and references therein). However, early-type stars have radiative envelopes, or very thin convective layers close to the surface due to peaks in the opacities (e.g. Richard et al. 2001; Cantiello et al. 2009). These thin layers are thought to produce surface magnetic fields of the order of a few Gauss (Cantiello & Braithwaite 2019), but this is not sufficient to explain the observed large-scale magnetic fields of these stars. For example, chemically peculiar Ap/Bp stars, that are intermediate mass ( $1.5 - 6 M_{\odot}$ ) MS stars, host surface magnetic fields with mean strengths between 200 G and 30 kG (Shorlin et al. 2002; Aurière et al. 2007). In most cases the magnetic field is a simple dipole with its axis misaligned with the rota-

tion axis (Donati & Landstreet 2009; Kochukhov et al. 2015). Some Ap/Bp stars exhibit more complex geometries and multipoles are required to fit the data (e.g. Kochukhov & Wade 2016; Silvester et al. 2017). However, these multipolar fields seem to decay faster than the commonly observed dipolar fields (Shultz et al. 2019). As the origin of these large-scale magnetic fields cannot be explained via convective envelope dynamo, several theories have been proposed, such as a dynamo operating on the radiative layers as a result of the interaction of a magnetic instability and differential rotation. An example is the Tayler-Spruit dynamo scenario (Spruit 2002; Petitdemange et al. 2023, 2024).

However, the simple topologies and the stability over long timescales of the observed magnetic fields seem to support the idea of fossil fields, that is, magnetic fields originating from an earlier stage of stellar evolution (Alecian et al. 2019). In the pre-MS evolution, Herbig Ae/Be stars (Waters & Waelkens 1998; Waters 2006) are believed to be the predecessors of MS Ap/Bp stars, and although their average magnetic fields are substantially weaker than those from Ap/Bp stars (see e.g. Hubrig et al. 2004; Kholtygin et al. 2019; Hubrig et al. 2020), they still share some common characteristics, such as their incidence rate (Hubrig et al. 2011; Alecian et al. 2013a,b). This suggests that the magnetic field is already present in the pre-MS stage, evolving from the molecular cloud to the MS (Moss 2003; Schleicher et al. 2023). It has also been proposed that the strong observed magnetic field could be the result of a merger with another protostar close to the end of the formation process (e.g. Ferrario et al.

2009; Schneider et al. 2019). In any case, the thick radiative envelope of Ap/Bp stars might provide the ideal conditions for the fossil magnetic field to reach a stable equilibrium and evolve on a diffusive timescale. Cowling (1945) realized that in the radiative core of the Sun, this timescale is of the order of  $10^{10}$  years, therefore, a magnetic field in equilibrium could survive for the entire MS lifetime of the star. However, finding stable configurations with analytical methods has been proven to be a challenging task. The energy method of Bernstein et al. (1958) shows that purely poloidal and purely toroidal fields in highly ideal conditions are unstable under perturbations (Taylor 1973; Markey & Tayler 1973; Wright 1973; Markey & Tayler 1974). Numerical MHD simulations by Braithwaite & Nordlund (2006) produced stable configurations in the radiative interior of a  $2 M_{\odot}$  A-type star starting from an initially random field, which relaxed to a stable roughly axisymmetric torus inside the star, with poloidal and toroidal components of comparable strength, leading to a roughly dipolar surface field. Non-axisymmetric configurations have also been found, starting from turbulent initial conditions (Braithwaite 2008). Recently, simulations by Becerra et al. (2022) showed that random magnetic field configurations seem to always evolve to a stable equilibrium in stably stratified environments, confirming previous results.

In addition to deep radiative envelopes, early-type stars have convective cores due to the temperature sensitivity of the CNO cycle. These cores have vigorous convective motions and differential rotation that can lead to dynamo action (Browning et al. 2004). Indeed, strong core dynamos have been found in numerical 3D simulations (Brun et al. 2005; Augustson et al. 2016; Hidalgo et al. 2024). Recently, an upper limit of  $B_r \approx 500$  kG was estimated for the B star HD 43317 by Lecoanet et al. (2022), based on its g-mode frequencies (Buysschaert et al. 2018). This limit seems to support the idea of a strong core dynamo inside these stars, due to the fact that this magnetic field strength would be hard to achieve exclusively with fossil fields. However, these magnetic fields are most likely unable to create large-scale structures on the stellar surface (Schuessler & Paehler 1978; Parker 1979; MacDonald & Mullan 2004), and only a very small percentage of the magnetic flux can be transported there (MacGregor & Cassinelli 2003; Hidalgo et al. 2024). Interestingly, a core dynamo can potentially interact with a fossil field (see Boyer & Levy 1984, in the context of the Sun). Featherstone et al. (2009) performed numerical simulations of a  $2 M_{\odot}$  A-type star core dynamo initially hosting near-equipartition fields surrounded by a fraction of the radiative envelope. The inclusion of a fossil field with poloidal and toroidal components led to a superequipartition state where magnetic energy is roughly ten times stronger than kinetic energy. This might have some interesting implications, for example, if this magnetic field is strong enough, then a larger magnetic flux can possibly be transported to the surface by magnetic buoyancy (e.g. MacDonald & Mullan 2004). Furthermore, how an enhanced core dynamo affects the fossil field at the stellar surface and inside the radiative envelope is unknown.

Simulations studying the stability of magnetic fields assume a stably stratified environment and neglect the presence of a core dynamo (e.g. Braithwaite & Spruit 2004; Becerra et al. 2022). In the current study, we impose a fossil field into a star-in-a-box setup of a MS A-type star hosting a core dynamo presented in Hidalgo et al. (2024); hereafter Paper I. This setup allows us to study how different fossil fields affect the core dynamo and how the surface magnetic field is modified. Another issue to be explored, is how the inclination of the fossil field affects the results, as in most of the Ap stars the axes of the magnetic dipole and rotation do not coincide. The models and methods are ex-

plained in Sect. 2. The initial state of the core dynamo and the explored fossil fields configurations are described in Sect. 3. The results of the simulations are discussed in Sect. 4. Summary and conclusions of the study are presented in Sect. 5.

## 2. Numerical models

### 2.1. A-type star setup

The setup used here is the same as that in Paper I, which is based on the star-in-a-box setup from Käpylä (2021). The star has a radius  $R$  and it is embedded into a cube of side  $l = 2.2R$ , where the Cartesian coordinates range from  $-l/2$  to  $l/2$ . The convective core encompasses 20% of the radial extent of the star, while the rest is stably stratified (see Paper I, for more details). The fully compressible set of MHD equations is:

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} - \eta \mu_0 \mathbf{J}, \quad (1)$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{U}, \quad (2)$$

$$\frac{D U}{Dt} = -\nabla \Phi - \frac{1}{\rho} (\nabla p - \nabla \cdot 2\nu \rho \mathbf{S} + \mathbf{J} \times \mathbf{B}) - 2\boldsymbol{\Omega} \times \mathbf{U} + \mathbf{f}_d, \quad (3)$$

$$T \frac{D s}{Dt} = -\frac{1}{\rho} [\nabla \cdot (\mathbf{F}_{\text{rad}} + \mathbf{F}_{\text{SGS}}) + \mathcal{H} - C + \mu_0 \eta \mathbf{J}^2] + 2\nu \mathbf{S}^2, \quad (4)$$

where  $\mathbf{A}$  is the magnetic vector potential,  $\mathbf{U}$  is the flow velocity,  $\mathbf{B} = \nabla \times \mathbf{A}$  is the magnetic field,  $\eta$  is the magnetic diffusivity,  $\mu_0$  is the magnetic permeability of vacuum,  $\mathbf{J} = \nabla \times \mathbf{B} / \mu_0$  is the current density,  $D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla$  is the advective derivative,  $\rho$  is the mass density,  $p$  is the pressure,  $\Phi$  is the fixed gravitational potential obtained from a 1D model,  $\nu$  is the kinematic viscosity,  $\mathbf{S}$  is the traceless rate-of-strain tensor with:

$$S_{ij} = \frac{1}{2} (\partial_j U_i + \partial_i U_j) - \frac{1}{3} \delta_{ij} \nabla \cdot \mathbf{U}, \quad (5)$$

where  $\delta_{ij}$  is the Kronecker delta. The rotation vector is  $\boldsymbol{\Omega} = \Omega_0 \hat{\mathbf{z}}$  and  $\mathbf{f}_d$  damps the flows outside the star, with

$$\mathbf{f}_d = -\frac{\mathbf{U}}{\tau_{\text{damp}}} f_e(r), \quad (6)$$

where  $\tau_{\text{damp}} = 0.2\tau_{\text{ff}} \approx 1.5$  days is the damping timescale,  $\tau_{\text{ff}} = \sqrt{R^3/GM}$  is the freefall time, and

$$f_e(r) = \frac{1}{2} \left( 1 + \tanh \frac{r - r_{\text{damp}}}{w_{\text{damp}}} \right), \quad (7)$$

where  $r = \sqrt{x^2 + y^2 + z^2}$ ,  $r_{\text{damp}} = 1.03R$  is the radius where the damping starts, and where  $w_{\text{damp}} = 0.02R$  is its width.  $T$  is the temperature,  $s$  is the specific entropy,  $\mathbf{F}_{\text{rad}}$  is the radiative flux

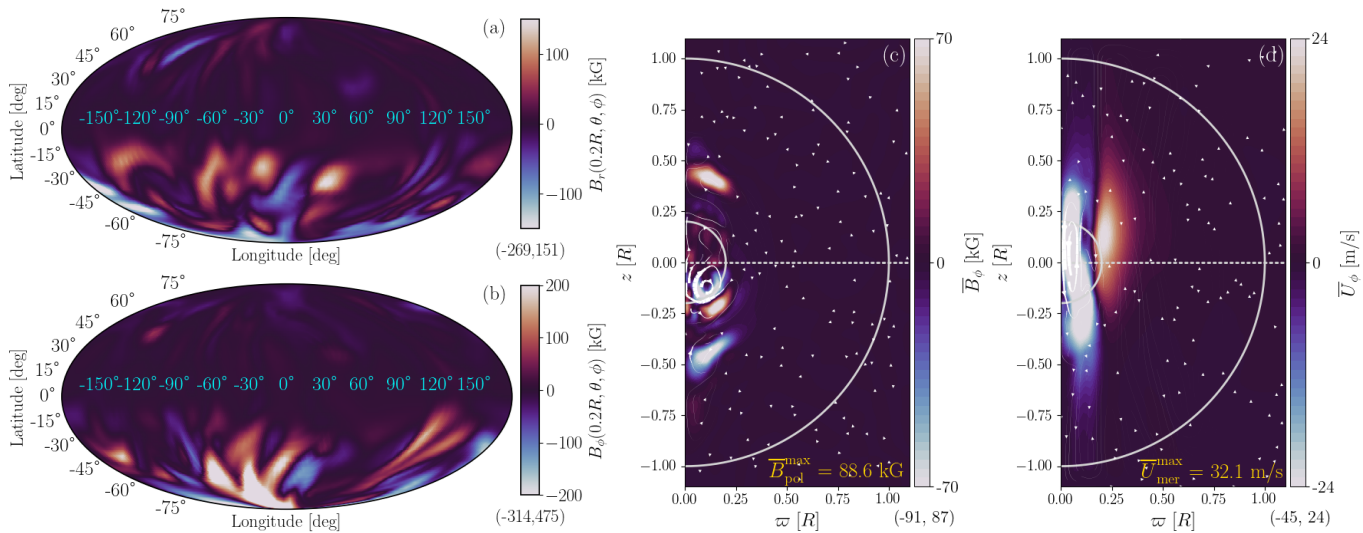
$$\mathbf{F}_{\text{rad}} = -K \nabla T, \quad (8)$$

where  $K$  is the heat conductivity, and  $\mathbf{F}_{\text{SGS}}$  is the sub-grid-scale (SGS) entropy flux

$$\mathbf{F}_{\text{SGS}} = -\chi_{\text{SGS}} \rho \nabla s', \quad (9)$$

where  $\chi_{\text{SGS}}$  is the SGS diffusion coefficient,  $s' = s - \langle s \rangle_t$  is the fluctuating entropy, and  $\langle s \rangle_t(\mathbf{x})$  is a temporal mean of the specific entropy. The heating function  $\mathcal{H}$  follows

$$\mathcal{H}(r) = \frac{L_{\text{sim}}}{(2\pi w_L^2)^{3/2}} \exp\left(-\frac{r^2}{2w_L^2}\right), \quad (10)$$



**Fig. 1.** Magnetic fields and flows from a snapshot of MHDr2 at  $t = 85$  yrs. Panels (a), (b): Radial and toroidal magnetic fields at the surface of the convective zone ( $r = 0.2R$ ). Panel (c): Azimuthally averaged toroidal (colormap) and poloidal (arrows) magnetic fields. Panel (d): Azimuthally averaged toroidal flows (colormap) and meridional circulation (arrows). All the panels are clipped for a better display, and the minimum and maximum values are indicated below the colorbar.

which is a normalized Gaussian that parameterize the nuclear energy production in the core of the star, where  $L_{\text{sim}}$  is the luminosity in the simulation, and  $w_L = 0.1R$  is the width of the Gaussian. The cooling function  $C(\mathbf{x})$  is given by

$$C(\mathbf{x}) = \rho c_P \frac{T(\mathbf{x}) - T_{\text{surf}}}{\tau_{\text{cool}}} f_e(r), \quad (11)$$

which models radiative losses above the stellar surface, where  $c_P$  is the heat capacity at constant pressure,  $T_{\text{surf}} = T(R)$  is the temperature at the stellar surface,  $\tau_{\text{cool}} = \tau_{\text{damp}}$  is a cooling timescale, and  $f_e(r)$  is given by Eq. (7), with  $r_{\text{cool}} = r_{\text{damp}}$  and  $w_{\text{cool}} = w_{\text{damp}}$ . The ideal gas equation of state  $p = (c_P - c_V)\rho T$  is assumed, where  $c_V$  is the heat capacity at constant volume. The model has radial profiles for the diffusivities  $\nu$  and  $\eta$ , where radiative zones have values that are  $10^2$  times smaller than in convective zones (Käpylä 2022). Furthermore, sixth-order hyperdiffusivity terms are added in the dynamical equations (see e.g. Brandenburg & Sarson 2002; Johansen & Klahr 2005; Lyra et al. 2017).

The simulations were run with the PENCIL CODE<sup>1</sup> (Pencil Code Collaboration et al. 2021), which is a highly modular high-order finite-difference code for solving ordinary and partial differential equations, on a grid of  $200^3$  equally distributed grid points.

## 2.2. Units and relation to reality

The units of length, time, density, entropy and magnetic field are given by

$$[x] = R, [t] = \tau_{\text{ff}}, [\rho] = \rho_0, [s] = c_P, [B] = \sqrt{\mu_0 \rho_0} [x] / [t]. \quad (12)$$

In this study, the quantities are typically expressed in physical units. The conversion factors are the same as those given by Eqs. (17) and (18) in Paper I. The stellar parameters were obtained from a 1D MESA model with  $2.2 M_{\odot}$  (Paxton et al. 2019), and are the same as those from Paper I, i.e., the radius, mass density and temperature of the stellar center, and stellar luminosity are  $R_{\star} = 2.1 R_{\odot}$ ,  $\rho_0 = 5.5 \cdot 10^4 \text{ kg m}^{-3}$ ,  $T_0 = 2.3 \cdot 10^7 \text{ K}$ ,

$L_{\star} = 23.5 L_{\odot}$ , respectively. Furthermore, as we are using the fully compressible formulation of the MHD equations, the enhanced luminosity approach is also used in these simulations (see Dobler et al. 2006; Käpylä et al. 2020; Käpylä 2021, for a detailed justification). To have a consistent rotational influence on the flow, the rotation rate is enhanced following the recipe in Appendix A of Käpylä et al. (2020).

## 2.3. Diagnostics quantities

The rotational influence on the flow is measured by the global Coriolis number,

$$\text{Co} = \frac{2\Omega_0}{u_{\text{rms}} k_R}, \quad (13)$$

where  $u_{\text{rms}}$  is the root-mean-square (rms) velocity averaged over the convection zone and  $k_R = 2\pi/\Delta r$  is an estimate of the largest convective eddies in the system, where  $\Delta r = 0.2R$  is the depth of the convective zone. The fluid and magnetic Reynolds numbers, and the SGS Péclet number are defined as:

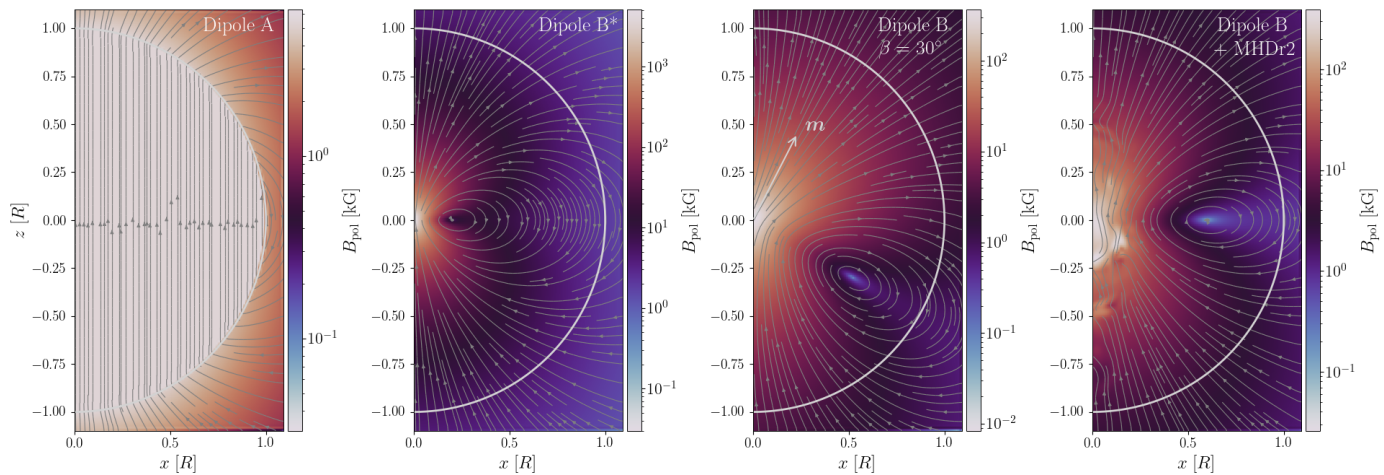
$$\text{Re} = \frac{u_{\text{rms}}}{\nu k_R}, \quad \text{Re}_M = \frac{u_{\text{rms}}}{\eta k_R}, \quad \text{Pe} = \frac{u_{\text{rms}}}{\chi_{\text{SGS}} k_R}. \quad (14)$$

## 3. Imposing a magnetic field

### 3.1. Initial state of the simulation

To analyze how an imposed fossil field affects the core dynamo, the fossil field configurations are added to snapshots of an already saturated core dynamo simulation. We chose run MHDr2 from Paper I, which hosts a dynamo with the largest rotation period ( $P_{\text{rot}} = 15$  days) and therefore, the smallest Coriolis number ( $\text{Co} = 10.1$ ) in that study. The core of this simulation ( $r < 0.2R$ ) has a root-mean-square (rms) magnetic field of  $B_{\text{rms}} = 60 \text{ kG}$ , solar-like differential rotation, and a hemispherical dynamo with most of the activity located on its southern hemisphere. The snapshot where the fossil field was imposed corresponds to a simulated time of  $t = 4135\tau_{\text{ff}}$ , or 85 years in physical units. In Fig. 1(a,b) the radial and toroidal magnetic fields at the surface of the convective zone ( $r = 0.2R$ ) from this snapshot are shown.

<sup>1</sup> <https://pencil-code.org/>



**Fig. 2.** Imposed initial (fossil) magnetic fields in our simulations. The arrows represent the poloidal magnetic field lines, and the colormap the intensity of this component. Dipole A and Dipole B\* are aligned to the rotational axis ( $\beta = 0^\circ$ ), and as an example of a misaligned dipole an inclination of  $\beta = 30^\circ$  was added in Dipole B. The last panel corresponds to the initial snapshot of run DipB (Dipole B + MHD r2).

Both magnetic field components are highly concentrated on the southern hemisphere. Furthermore, the amplitude of the toroidal component is larger than that of the radial component. Panel (c) displays the azimuthally averaged magnetic field in the entire box. The hemispheric nature of the core dynamo is still clearly visible from both components. However, strong magnetic fields are also present at the bottom radiative envelope. This is most likely due to flows penetrating into the radiative layer near the rotation axis. Finally, on panel (d) the azimuthally averaged flows are shown. The flows penetrating deeply into the radiative envelope are most likely due to lower Brunt-Väisälä frequency (for example, see Fig. 5 of Korre & Featherstone 2024), and therefore, lower Richardson numbers related to rotation than in real stars (e.g. Käpylä 2024) achieved in our simulations. These flows also transport some of the magnetic flux from the core dynamo to the stellar surface, leading to an rms surface magnetic field of  $\sim 0.1$  kG at the poles and as low as  $10^{-5}$  kG elsewhere. MHD r2 has subequipartition magnetic fields, where the ratio of the magnetic to kinetic energies is  $E_{\text{mag}}/E_{\text{kin}} = 0.196$  in the convective core ( $r < \Delta r$ ), and 0.322 in the entire star ( $r < R$ ).

### 3.2. Dipolar fossil field

The majority of Ap stars exhibit large-scale poloidal magnetic fields. For example, in HD 75049, 90% of magnetic energy is concentrated in the dipolar  $\ell = 1$  spherical harmonic mode with a polar strength of  $B_p = 36$  kG and an obliquity of  $36^\circ$  (Kochukhov et al. 2015). Similarly, in HD 24712, 96% of the magnetic energy is concentrated in  $\ell = 1$  (Rusomarov et al. 2015). Therefore, we choose to impose a purely poloidal ( $B_\phi = 0$ ) fossil field, whose vector potential is

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{r}}{r^3}, \quad (15)$$

where  $\mathbf{m}$  is the magnetic dipole moment. As discussed, the observed magnetic fields of Ap/Bp stars are typically misaligned with their rotation axis. Therefore, we define  $\mathbf{m}$  as

$$\mathbf{m}(\beta) = m_0(\sin\beta\hat{x} + \cos\beta\hat{z}), \quad (16)$$

where  $\beta$  is the obliquity angle of the magnetic dipole with respect to  $\Omega$ , and  $m_0$  is the initial dipolar amplitude. Thus, Eq. (15) can

be expressed as

$$\mathbf{A}(\mathbf{r}, \beta) = \frac{\mu_0 m_0}{4\pi} \left( -\frac{y \cos\beta}{r^3} \hat{x} + \frac{x \cos\beta - z \sin\beta}{r^3} \hat{y} + \frac{y \sin\beta}{r^3} \hat{z} \right). \quad (17)$$

The sample of Ap stars presented by Landstreet & Mathys (2000) shows that slow rotators ( $P_{\text{rot}} > 25$  days) have small obliquity angles, with an excess around  $20^\circ$ , while fast rotators tend to have larger values of  $\beta$ . More recently, Shultz et al. (2019) reported that the distribution of  $\beta$  seems to be statistically consistent with a random distribution, which fits with the data of Aurière et al. (2007), and Sikora et al. (2019, see their Fig. 11). In this study, we consider  $0^\circ \leq \beta \leq 90^\circ$ . More specifically,  $\beta = 0^\circ$  (0),  $30^\circ$  ( $\pi/6$ ),  $60^\circ$  ( $\pi/3$ ) and  $90^\circ$  ( $\pi/2$ ), to explore the entire range of inclinations. We further test  $\beta = 80^\circ$  ( $4\pi/3$ ) and  $85^\circ$  ( $17\pi/36$ ) in one specific case, to study the transition to a horizontal dipole in more detail.

We define two dipolar initial conditions, the first one (Dipole A) corresponding to the magnetic field of a magnetized sphere, that is, a uniform magnetic field inside ( $r < R$ ), matched by a dipole field in the exterior ( $r > R$ ), while the second configuration corresponds to a dipole field in the entire domain. Eq. (17) has a singularity for  $r \rightarrow 0$ , and therefore we added a constant  $\epsilon$  to  $r$ , such that  $r' \equiv r + \epsilon$  is used in the calculation of the dipole fields. Two values of  $\epsilon$  were chosen, where  $\epsilon_1 < \epsilon_2$ .  $\epsilon_1$  ( $\epsilon_2$ ) produces a  $B_{\text{rms}}^{\text{pol}}$  of roughly 500 kG (100 kG) in the core,  $r < 0.2R$ , and 48 kG (14 kG) in the entire star. The runs with  $\epsilon_1$  are labeled as Dipole B\* and those with  $\epsilon_2$  as Dipole B. The initial value of  $m_0$  was chosen such that all the mentioned magnetic configurations have a dipolar amplitude of roughly 6 kG at the stellar surface, which is typical from Ap/Bp stars (Aurière et al. 2007; Shultz et al. 2019). All of the initial conditions are displayed in the first three panels of Figure 2. An obliquity angle of  $30^\circ$  was added in Dipole B (third panel) as an example of a tilted dipole initial condition.

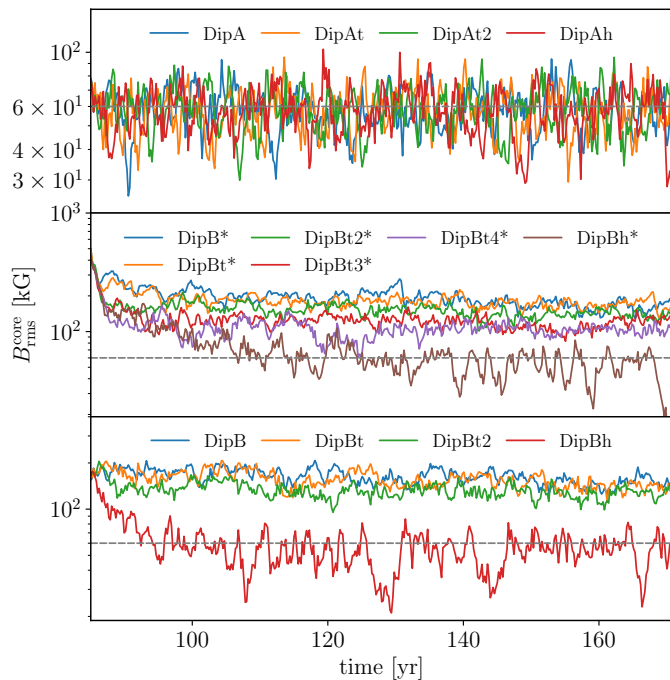
## 4. Results

The simulations are divided into three sets, the names of which reflect the chosen initial configuration. For example, runs with Dipole B are labeled as DipB. The simulations, as well as the diagnostic quantities, are listed in Table 1.

**Table 1.** Summary of the simulations.

Run	$\beta$ [°]	$u_{\text{rms}}$ [m/s]	$(B_{\text{rms}}^{\text{core}}, B_{\text{rms}}^{\text{star}})$ [kG]	$(B_{\text{pol}}^{\text{core}}, B_{\text{tor}}^{\text{core}})$ [kG]	Co	Pe	Re	Re <sub>M</sub>	$P_{\text{cyc}}$ [years]
MHDr2	-	51	(60, 21)	(19, 22)	10.1	7	35	24	$1.88 \pm 0.09$
DipA	0°	50	(60, 33)	(23, 19)	8.7	8	40	28	$1.66 \pm 0.07$
DipAt	30°	50	(58, 31)	(22, 19)	8.7	8	40	28	$1.79 \pm 0.03$
DipAt2	60°	51	(59, 28)	(20, 21)	8.7	8	40	28	$1.81 \pm 0.05$
DipAh	90°	53	(58, 23)	(18, 21)	8.3	8	42	29	$1.88 \pm 0.03$
DipB*	0°	28	(172, 53)	(102, 54)	15.6	4	22	15	no cycles
DipBt*	30°	28	(172, 53)	(96, 65)	15.8	4	22	15	no cycles
DipBt2*	60°	31	(141, 44)	(77, 45)	14.2	5	24	17	no cycles
DipBt3*	80°	35	(116, 37)	(54, 32)	12.4	5	28	19	no cycles
DipBt4*	85°	36	(105, 34)	(44, 27)	11.9	6	29	20	no cycles
DipBh*	90°	52	(53, 26)	(16, 16)	8.4	8	42	29	$1.71 \pm 0.26$
DipB	0°	29	(157, 47)	(90, 52)	15.2	4	23	16	no cycles
DipBt	30°	30	(147, 46)	(83, 49)	14.5	5	24	17	no cycles
DipBt2	60°	33	(129, 40)	(64, 35)	13.3	5	26	18	no cycles
DipBh	90°	53	(55, 26)	(18, 20)	8.5	8	41	29	$1.66 \pm 0.04$

**Notes.** From left to right: obliquity angle of the added dipole field, the rms velocity  $u_{\text{rms}}$ , the volume-averaged rms magnetic field  $B_{\text{rms}}$ , the poloidal and toroidal components of  $B_{\text{rms}}$ , the Coriolis number, the SGS Péclet number, the fluid and magnetic Reynolds numbers, and the magnetic cycle period. Quantities are averaged over the convective core  $r < 0.2R$  unless “star” is indicated in which case the average is over  $r < R$ .



**Fig. 3.** Temporal evolution of the rms magnetic field in the convective core ( $r < 0.2R$ ) from all simulations. The gray dashed line indicates 60 kG, which is the saturated value of the core dynamo from MHDr2.

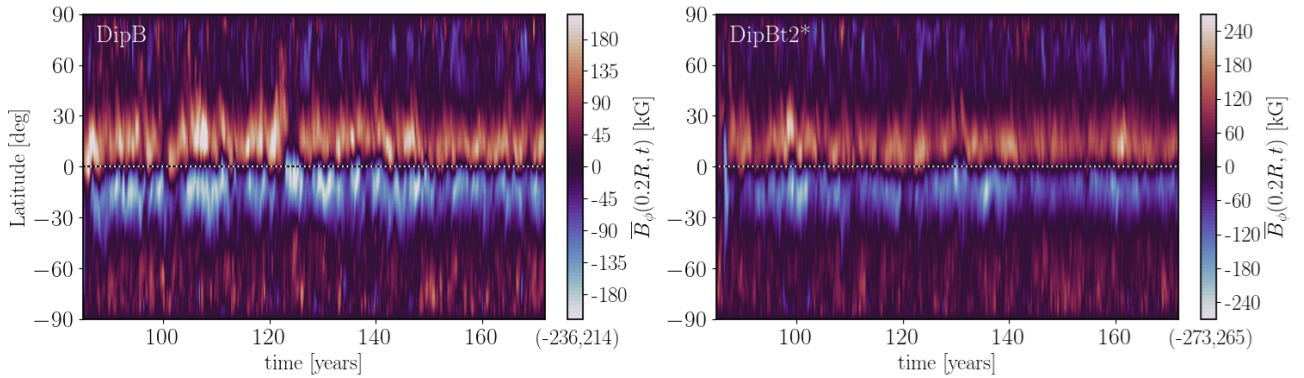
#### 4.1. Core dynamos

All of the current simulations host a strong core dynamo. However, in some cases the dynamo is enhanced by the fossil field while in others it remains mostly unaffected (see Table 1). More specifically, runs with Dipole A have essentially the same  $B_{\text{rms}}^{\text{core}}$  as the run with no added dipole (MHDr2), irrespective of the inclination angle (see upper panel of Figure 3). On the other hand, runs with Dipoles B and B\* have stronger core magnetic fields  $B_{\text{rms}}^{\text{core}}$ , in some cases thrice that of MHDr2. Interestingly, in the latter two sets there is a relation between the strength of the magnetic fields and the obliquity angle  $\beta$ .

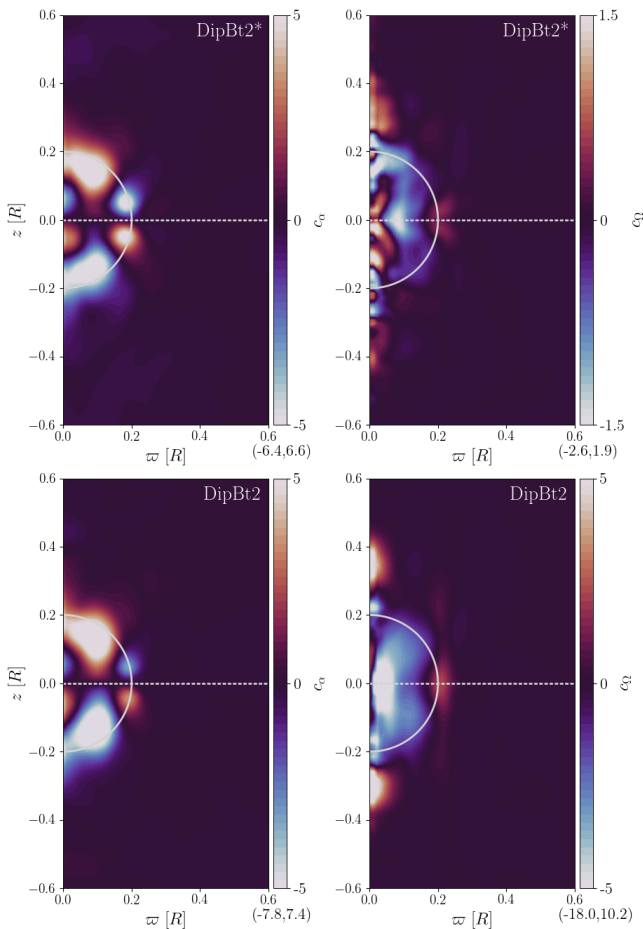
##### 4.1.1. Enhanced dynamos

Figure 3 shows that runs with Dipoles B and B\* start with the same core magnetic field amplitudes, and that only the cases with  $\beta = 90^\circ$  decay to the same level as the core dynamo without any additional dipole field (the gray dashed line). The aligned cases (DipB, DipB\*) and the rest of inclinations (DipBt, DipBt2, DipBt\*, DipBt2\*, DipBt3\*, DipBt4\*) saturate in higher values than those of run MHDr2, suggesting enhanced core dynamo action. The saturation levels of these runs are similar although temporal averages (fourth column of Table 1) show that there is an inverse relation between  $\beta$  and  $B_{\text{rms}}$ . The strongest rms magnetic fields for both averages, are obtained when the dipole is aligned with the rotation axis. In the misaligned cases the field decreases with increasing  $\beta$ . The only exception to this trend is DipBt\*, which has the same averaged flow velocity and rms magnetic fields as DipB\*. Interestingly, inside the core, DipBt\* has even a stronger toroidal component than DipB\*, while in the rest of runs the inverse relation applies for both poloidal and toroidal components. Furthermore, weaker magnetic fields leads to weaker quenching of flows. It is worth mentioning that although we only added a strong poloidal component in the simulations, the toroidal component of the core dynamo also increases in the enhanced cases. The cases with horizontal dipoles decayed almost completely. These runs will be analyzed separately in Section 4.1.2.

The azimuthally averaged toroidal magnetic fields at  $r = 0.2R$  as a function of time and latitude for representative enhanced cases are shown in Fig. 4. The initially cyclic and hemispheric solutions (see Fig. 2 of Paper I) are replaced by quasi-stationary solutions. All the simulations with an enhanced core dynamo end up in the same configuration, irrespective of the obliquity angle. Quasi-stationary dynamo solutions appear in simulations either with low Coriolis numbers (e.g. Käpylä et al. 2013; Strugarek et al. 2018; Brun et al. 2022; Ortiz-Rodríguez et al. 2023), or in the strong field branch at rapid rotation (e.g. Christensen & Aubert 2006; Gastine et al. 2012; Yadav et al. 2015). The current simulations land in the latter due to the strong added dipole fields. The obtained configuration resembles a dipole, with a positive polarity in latitudes from  $0^\circ$  to  $30^\circ$  and



**Fig. 4.** Time-latitude diagrams of the azimuthally averaged toroidal magnetic field  $\bar{B}_\phi(r = 0.2R, \theta, t)$  of runs DipB and DipBt2\*. The run label is indicated in the upper left corner of each panel.



**Fig. 5.** Dynamo parameters  $c_\alpha = \alpha \Delta r / \eta_{\text{turb}}$  and  $c_\Omega = \partial \bar{\Omega} / \partial r (\Delta r)^3 / \eta_{\text{turb}}$  from DipBt2\* (upper panels) and DipBt2 (bottom panels). All the panels are clipped and cropped (to  $0.6R$ ) for better legibility and comparison.

negative polarity from  $0^\circ$  to  $-30^\circ$ . Beyond these latitudes, the polarity reverses, and the magnetic fields are weaker than in the latitudes close to the equator. As the added dipole field is purely poloidal, the toroidal fields likely come from a dynamo process in the convective core.

To understand the origin of these dynamos, we make use of dynamo numbers based on mean-field dynamo theory (Krause & Rädler 1980; Brandenburg & Subramanian 2005). To quantify

the  $\alpha$  and  $\Omega$  effects, it is useful to define (Käpylä et al. 2013)

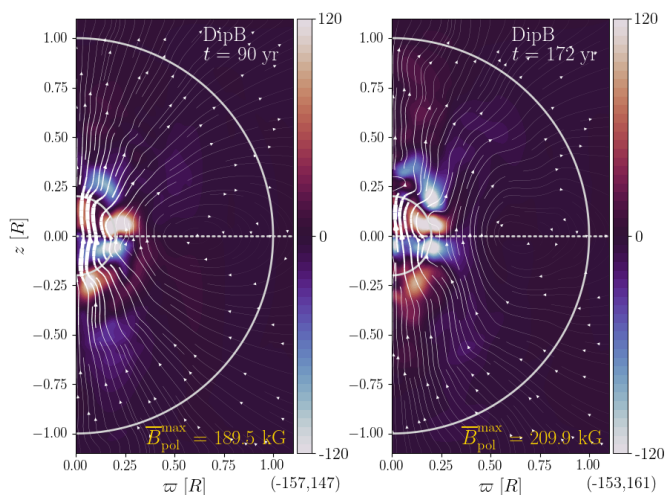
$$c_\alpha = \frac{\alpha \Delta r}{\eta_{\text{turb}}}, \quad c_\Omega = \frac{\partial \bar{\Omega} / \partial r (\Delta r)^3}{\eta_{\text{turb}}}, \quad (18)$$

where  $\eta_{\text{turb}} = \tau u_{\text{rms}}^2 / 3$  is an estimate of the turbulent diffusivity,  $\tau = \Delta r / u_{\text{rms}}$  is the convective turnover time, and  $\bar{\Omega}$  is the time- and azimuthally averaged rotation rate. The non-linear  $\alpha$  effect is proportional to the difference between the fluctuating kinetic and magnetic helicities, given by (Pouquet et al. 1976)

$$\alpha = -\frac{\tau}{3} [(\overline{U \cdot \omega} - \overline{U} \cdot \overline{\omega}) - (\overline{J \cdot B} - \overline{J} \cdot \overline{B}) / \bar{\rho}], \quad (19)$$

where  $\omega = \nabla \times U$  is the vorticity. The dynamo parameters (18) from DipBt2 and DipBt2\* are shown in Figure 5. The profiles and amplitudes of  $c_\alpha$  are similar to those of runs without a fossil field (see Fig. 3 of Paper I). However, the current profiles show zones of strong  $\alpha$  effect with a different sign close to the equator and the surface of the convective core. These are also the locations where the magnetic field is the strongest (see Figure 4). The profile of  $c_\alpha$  is antisymmetric with respect to the equator. The profiles of  $c_\Omega$  are of opposite sign in comparison to those of runs in Paper I. This is a consequence of the anti-solar differential rotation of the core (see Section 4.3). However, the amplitudes of  $c_\Omega$  are much weaker than in Paper I. This is most clearly seen in runs with Dipole B\*, as can be seen from the profile of  $c_\Omega$  of DipBt2\*; see Fig. 5. The inclusion of a strong imposed field quenches the differential rotation, reducing the shear, which is translated in lower values of  $c_\Omega$ . Table 2 shows that the energy in differential rotation in the core and in the radiative envelope is much weaker than in MHDr2. Furthermore, in the enhanced dynamos, the poloidal energy is stronger than the toroidal energy, being typically  $E_{\text{mag}}^{\text{pol}} \approx 3 E_{\text{mag}}^{\text{tor}}$ .

We interpreted the core dynamos from Paper I as  $\alpha\Omega$  or  $\alpha^2\Omega$  dynamos, primarily because the signs of  $\alpha$ ,  $\partial_r \bar{\Omega}$  and the propagation direction of the dynamo waves and secondarily because  $c_\Omega \gg c_\alpha$  and the dominance of toroidal over poloidal fields. In the enhanced dynamos, the lower values of  $c_\Omega$ , weaker than  $c_\alpha$  in some runs, and the ratio  $E_{\text{mag}}^{\text{pol}} / E_{\text{mag}}^{\text{tor}} \approx 3$  suggest an  $\alpha^2$  dynamo. These dynamos rely on the  $\alpha$  effect, and no shear from differential rotation is directly needed. Their initial formulation suggested only non-oscillatory dynamos, for example, the axisymmetric models by Steenbeck & Krause (1969), which fits with the quasi-stationary solutions of the axisymmetric toroidal magnetic field found in our simulations. However, oscillatory solutions have also been found in numerical simulations (Baryshnikova & Shukurov 1987; Mitra et al. 2010; Käpylä et al. 2013;



**Fig. 6.** Azimuthally averaged toroidal magnetic field  $\bar{B}_\phi(\varpi, z)$  of DipB. The poloidal magnetic field is represented with arrows, where the width is proportional to the strength of the field. The values of  $\bar{B}_\phi$  are clipped and the maximum and minimum values ( $\bar{B}_\phi^{\min}$ ,  $\bar{B}_\phi^{\max}$ ) are indicated below the colorbar. The dashed line at  $z = 0$  represents the equator. The complete temporal evolution is available as an online movie.

Masada & Sano 2014) and analytical work (Raedler & Braeuer 1987; Brandenburg 2017).

The meridional distribution of the azimuthally averaged magnetic field of DipB is shown in Figure 6. In an early stage of the simulation, five years ( $\sim 240\tau_{\text{ff}}$ ) after the inclusion of the fossil field, the core dynamo already exhibits the described quasi-stationary behavior (see left panel). Furthermore, some of the toroidal magnetics are seen outside of the convective core at the base of the radiative envelope. The poloidal magnetic field follows the structure of Dipole B outside the core, while within the field is nearly vertical like in Dipole A. The magnetic field in the radiative zone is different than that generated by the core dynamo without an added dipole (see Fig. 4 of Paper I). Initially, the magnetic field spread close to the rotation axis from  $z \approx 0.3R$  to  $r \approx 0.5R$ , which is the location of the magnetic diffusivity jump. In Paper I we mention that this magnetic field is most likely transported by vertical flows (see, for example, the rightmost panel of Fig. 1), after which it evolves on long timescales compared to the period of the core dynamo due to the low diffusivities there. In the current simulations, there is much less magnetic fields beyond the radial jump of diffusivities ( $r \approx 0.35R$ ), indicating that in runs with the enhanced core dynamo the flows are not transporting magnetic field from the core to this zone as efficiently. The right panel of Figure 6 displays the magnetic field of DipB 82 years ( $\sim 3993\tau_{\text{ff}}$ ) after that in left panel. The quasi-stationary nature of the core dynamo remains unaffected. Furthermore, the dipolar structure present in most of the radiative envelope and at the stellar surface remains essentially unchanged, indicating that the enhanced core dynamo influences the surface poloidal magnetic field of the star only weakly (see Section 4.2).

Finally, these runs have superequipartition magnetic fields. More specifically, DipB\* and DipBt\* have  $E_{\text{mag}}/E_{\text{kin}} \approx 5$  in the convective core, and  $E_{\text{mag}}/E_{\text{kin}} \approx 10$  in the radiative envelope. The core dynamo of the fast rotators from Augustson et al. (2016) achieve similar regimes as those in our simulations, for example, their fastest rotator (M16) reaches  $E_{\text{mag}}/E_{\text{kin}} = 5.02$ . While still in superequipartition, it is unclear how  $E_{\text{mag}}/E_{\text{kin}}$  changes in the simulations of Featherstone et al. (2009) with

an added purely poloidal field. Nevertheless, the inclusion of a mixed field (with poloidal and toroidal components) increases the ratio in the core dynamo to  $E_{\text{mag}}/E_{\text{kin}} \approx 10$ . This ratio is comparable to what we find in the radiative envelope of our runs but larger than in our core dynamos. In the mixed case of Featherstone et al. (2009),  $E_{\text{mag}}/E_{\text{kin}} \approx 5.18$  in the radiative envelope (see their Table 1), which is less than in our simulations. This might be because they consider only a fraction of the radiative envelope. In runs with Dipole B, the enhanced dynamos are also in the superequipartition regime, but the ratios are somewhat smaller than those from runs with Dipole B\*, both in the convective core and in the radiative zone.

#### 4.1.2. The unaffected cases

The azimuthally averaged toroidal magnetic fields at  $r = 0.2R$  of representative simulations without an enhanced dynamo are shown in Figure 7. None of the core dynamos from runs with Dipole A got enhanced, and the obliquity angle of the dipole has no effect in the magnetic field configuration. This is likely a consequence of the strength of the dipole. While all the configurations have a surface dipolar amplitude of 6 kG, Dipole A keeps this amplitude constant inside the star, and therefore,  $B_{\text{rms}}^{\text{pol}}$  inside the core is  $\sim 6$  kG. This is much weaker than the fields from Dipoles B and B\* (see Section 3.2) or that from the core dynamo of MHDr2 ( $\sim 19$  kG). Furthermore, their magnetic cycle period is not affected either, being essentially the same as that from MHDr2 (see the last column of Table 1). Figure 7 shows that the core dynamo from DipA remains cyclic and hemispheric, occasionally changing its active hemisphere like run MHDr2\* of Paper I and in the fully convective model of Brown et al. (2020). All the runs with Dipole A show at least one dynamo migration except DipAh. One difference between these runs and MHDr2 is that close to the equator a weak structure similar to the quasi-stationary solutions of the enhanced dynamos is visible, except in DipAh. For example, this structure can be spotted in DipA where it is clearest between  $\sim 130$  and  $\sim 150$  years.

The right panel of Figure 7 display the toroidal magnetic field of the horizontal case ( $\beta = 90^\circ$ ) with Dipole B\*. In this case, the core dynamo exhibits the quasi-stationary solution described in Section 4.1.1 right after the inclusion of the dipole field. However, the solution is unstable and it decays after a few years, returning to the original hemispherical configuration of the core dynamo. The same happens in DipBh. Interestingly, this fast decay happens exclusively in the horizontal cases, whereas even the cases with  $\beta = 80^\circ$  and  $\beta = 85^\circ$  have a stable enhanced dynamo with at least twice the strength of the horizontal cases (see Table 1).

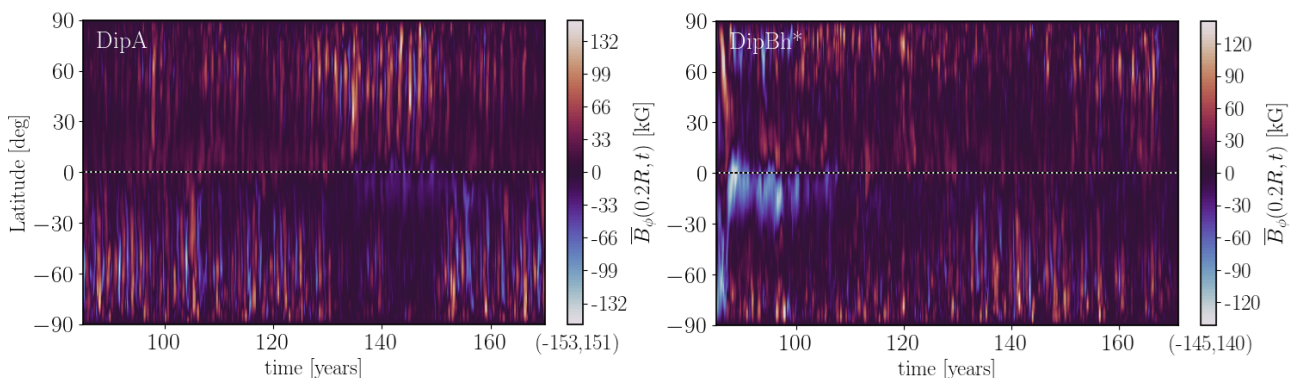
#### 4.2. Stability and surface magnetic fields

In principle, a purely poloidal magnetic field whose field lines are closed inside the star, like Dipole B and Dipole B\*, is unstable against adiabatic perturbations (Markey & Tayler 1973; Wright 1973; Markey & Tayler 1974). Similarly, a poloidal field with none of the field lines closed inside the star, like Dipole A, is unstable as well (Flowers & Ruderman 1977). However, these analytical results are highly idealized, as dissipative effects and rotation are not considered. Furthermore, we are not testing the stability of a purely poloidal magnetic field. In MHDr2, 37.8% of the magnetic energy inside the radiative envelope is toroidal, and as described in Section 4.1.1, the added poloidal field can also induce a toroidal component close to the core. Therefore,

**Table 2.** Kinetic and magnetic energies.

Run	$E_{\text{kin}} [10^{33}\text{J}]$	$E_{\text{kin}}^{\text{DR}}/E_{\text{kin}}$	$E_{\text{kin}}^{\text{MC}}/E_{\text{kin}}$	$E_{\text{mag}} [10^{33}\text{J}]$	$E_{\text{mag}}^{\text{tor}}/E_{\text{mag}}$	$E_{\text{mag}}^{\text{pol}}/E_{\text{mag}}$	$E_{\text{mag}}/E_{\text{kin}}$
<b>Convective core</b>							
MHDr2	8.88	0.346	0.026	1.74	0.140	0.097	0.196
DipA	8.60	0.059	0.025	1.72	0.109	0.143	0.200
DipAt	8.65	0.059	0.024	1.64	0.114	0.137	0.190
DipAt2	8.78	0.071	0.024	1.72	0.130	0.119	0.196
DipAh	9.48	0.062	0.028	1.66	0.137	0.096	0.176
DipB*	2.70	0.012	0.029	13.77	0.102	0.342	5.105
DipBt*	2.63	0.030	0.027	13.77	0.147	0.303	5.229
DipBt2*	3.24	0.008	0.021	9.34	0.104	0.290	2.879
DipBt3*	4.20	0.004	0.014	6.36	0.077	0.212	1.514
DipBt4*	4.57	0.015	0.014	5.19	0.071	0.173	1.135
DipBh*	9.27	0.063	0.025	1.37	0.100	0.088	0.147
DipB	2.84	0.006	0.025	11.41	0.112	0.326	4.018
DipBt	3.11	0.008	0.024	10.04	0.115	0.310	3.229
DipBt2	3.72	0.032	0.017	7.78	0.079	0.241	2.092
DipBh	9.72	0.178	0.027	1.48	0.134	0.104	0.152
<b>Radiative envelope</b>							
MHDr2	32.37	0.470	0.006	16.08	0.378	0.041	0.497
DipA	25.54	0.071	0.007	52.14	0.460	0.119	2.042
DipAt	25.65	0.061	0.007	46.38	0.427	0.074	1.808
DipAt2	27.28	0.088	0.006	37.99	0.376	0.092	1.393
DipAh	31.47	0.073	0.007	21.60	0.303	0.035	0.686
DipB*	9.00	0.018	0.005	89.82	0.165	0.220	9.983
DipBt*	9.47	0.047	0.005	91.13	0.211	0.170	9.628
DipBt2*	8.89	0.014	0.003	61.65	0.175	0.114	6.932
DipBt3*	10.49	0.005	0.003	46.78	0.108	0.047	4.460
DipBt4*	11.75	0.020	0.003	41.71	0.109	0.036	3.550
DipBh*	25.24	0.067	0.008	32.47	0.136	0.023	1.287
DipB	8.43	0.010	0.004	70.17	0.211	0.154	8.320
DipBt	8.74	0.011	0.004	68.53	0.184	0.125	7.840
DipBt2	9.24	0.045	0.003	52.21	0.145	0.074	5.651
DipBh	29.75	0.196	0.007	32.13	0.257	0.037	1.080

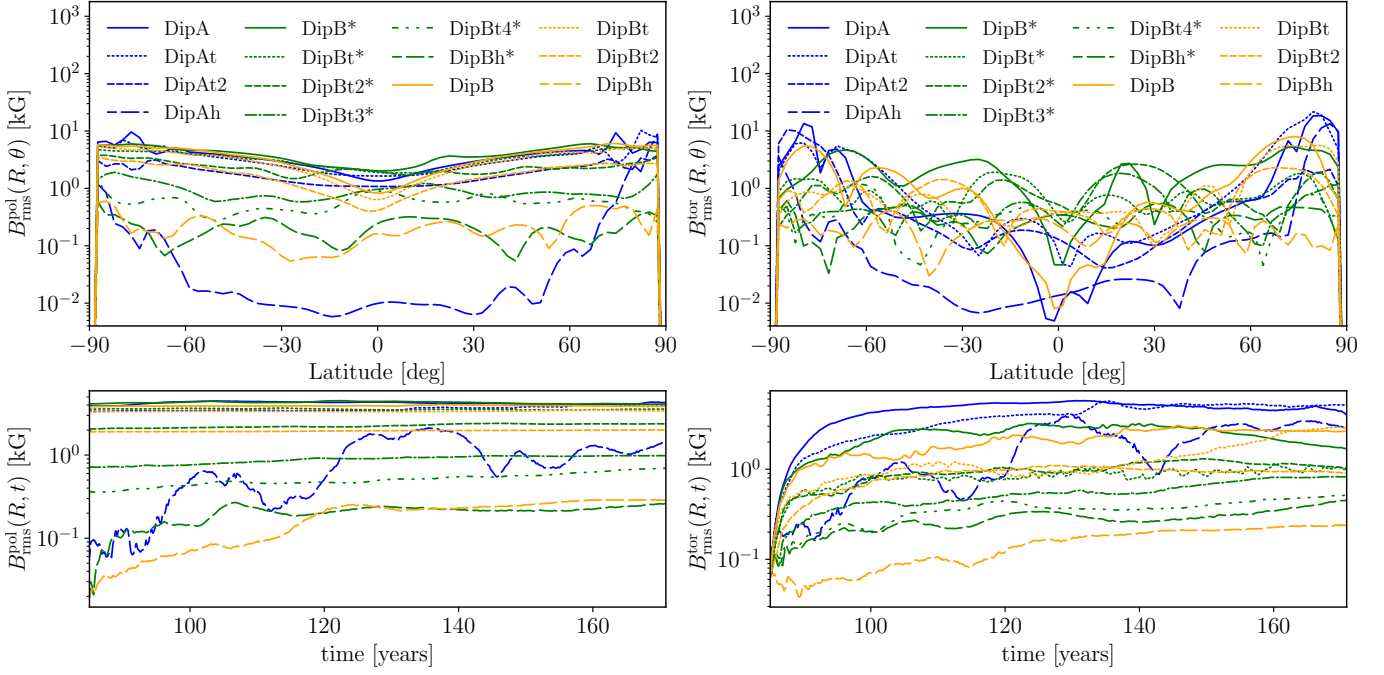
**Notes.** The total kinetic energy is  $E_{\text{kin}} = \frac{1}{2} \int \rho U^2 dV$ . The energies for the differential rotation (DR) and meridional circulation (MC), are  $E_{\text{kin}}^{\text{DR}} = \frac{1}{2} \int \rho \bar{U}_{\phi}^2 dV$  and  $E_{\text{kin}}^{\text{MC}} = \frac{1}{2} \int \rho (\bar{U}_{\sigma}^2 + \bar{U}_z^2) dV$ , respectively. The total magnetic energy is  $E_{\text{mag}} = \frac{1}{2\mu_0} \int \mathbf{B}^2 dV$ , and the toroidal and poloidal magnetic energies are  $E_{\text{mag}}^{\text{tor}} = \frac{1}{2\mu_0} \int \bar{B}_{\phi}^2 dV$  and  $E_{\text{mag}}^{\text{pol}} = \frac{1}{2\mu_0} \int (\bar{B}_{\sigma}^2 + \bar{B}_z^2) dV$ , respectively. The energies are averaged over time and integrated over the convective core and the radiative envelope. The total kinetic and magnetic energies are given in units of  $10^{33}\text{J}$ .


**Fig. 7.** Same as Figure 4, but for runs without an enhanced dynamo.

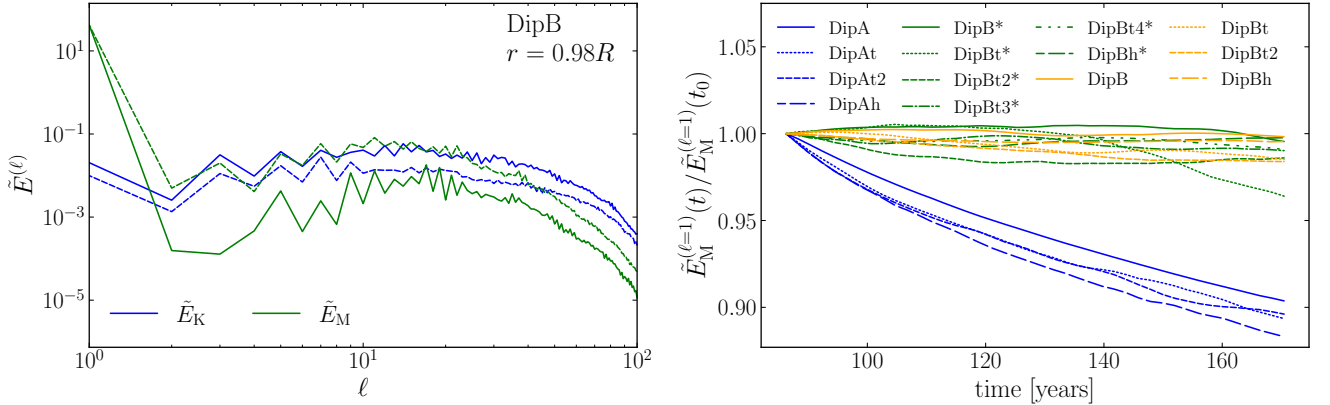
the radiative envelopes in our simulations have both components in all the cases, which as suggested by analytical estimations (Prendergast 1956; Wright 1973), and numerical results (Braithwaite & Spruit 2004; Braithwaite & Nordlund 2006), seems to be a necessary condition to achieve stability. Furthermore,

the stability condition reported by Braithwaite (2009) suggests  $10^{-5} \lesssim E_{\text{mag}}^{\text{pol}}/E_{\text{mag}} \lesssim 0.8$  for a MS A-type star, which is fulfilled inside the radiative envelope in all of our simulations (see column 7 of Table 2).





**Fig. 8.** Distribution and temporal evolution of the rms magnetic field at the stellar surface ( $r = R$ ) of all the simulations. *Upper panels:* Poloidal (left) and toroidal (right) components of the azimuthally and temporally averaged rms-field as a function of latitude. *Bottom panels:* Temporal evolution of the horizontally ( $\phi\theta$ ) averaged poloidal (left) and toroidal (right) rms-field.



**Fig. 9.** *Left panel:* Normalized power spectra of the velocity  $\tilde{E}_K^{(\ell)} = E_K^{(\ell)} / \sum_l E_K^{(\ell)}$  and magnetic fields  $\tilde{E}_M^{(\ell)} = E_M^{(\ell)} / \sum_l E_K^{(\ell)}$  from DipB at  $r = 0.98R$ . The solid lines show the power spectra from early times (first 15 years) of the simulation, and the dashed lines the late times (last 15 years). *Right panel:* Temporal evolution of the dipolar contribution ( $\ell = 1$ ) of the power spectra of the magnetic field  $\tilde{E}_M$  from all the simulations. The values were normalized by  $\tilde{E}_M^{(\ell=1)}(t_0)$  at the starting point of each simulation (at  $t_0 = 85$  years), for a better comparison.

The upper panels of Figure 8 show the latitudinal distribution of the poloidal (left) and toroidal (right) components of azimuthally and temporally averaged rms-field at the stellar surface. In the cases with  $\beta \neq 90^\circ$ , prominent poloidal fields are seen in the full range of latitudes with slightly weaker amplitudes around the equator. Interestingly, a toroidal component with roughly the same amplitude is also present in these runs. Distribution of  $B_\phi$  at the stellar surface is less coherent than the poloidal field, with significantly weaker values at the equator and maxima close to the poles. This is similar to the runs of Paper I (see their Fig. 6), although there the fields are much weaker ( $B_{\text{rms}} \sim 10^{-5}$  kG) apart from the poles. Temporal evolution of the horizontally averaged  $B_{\text{rms}}^{\text{pol}}$  is in the bottom left panel of Fig. 8. This component remains essentially unaffected in the runs with  $\beta \neq 0$ , while in the cases with  $\beta = 0$  it reaches saturation after  $\sim 120$  years in runs DipBh and DipBh\*. The case of DipAh is

different, because most of its activity is located in the poles, so the observed surface field is most likely a consequence of the core dynamo and the vertical flows, same as in Fig. 6 of Paper I, and not the fossil field that we are adding. From the bottom right panel it is visible that the toroidal component from all the runs is a consequence of the imposed fossil field, as it starts increasing right after its inclusion. In the runs with  $\beta \neq 0$  it grows until it roughly reaches the strength of the added dipole field. The origin of this toroidal component is not clear, but dynamo action in layers close to the surface seems highly unlikely, as the surface has almost no shear (see Section 4.3) and both dynamo parameters are close to zero. Therefore, this toroidal field is most likely transported from the enhanced core dynamo, or from the bottom radiative envelope. After the inclusion of Dipoles B and B\*, the advection timescales increased from 100 yrs (see Paper I) to 1000 and 640 yrs, respectively. However, turbulent diffu-

sion timescales are around 11 yrs in regions closer to the axis of rotation, and around 50 yrs in other latitudes. Therefore, the toroidal magnetic field is most likely transported to the surface due to turbulent diffusion.

The normalized power spectra of the velocity and magnetic field from a spherical harmonic decomposition (see Krause & Rädler 1980) of DipB is shown in the left panel of Fig. 9. The solid lines correspond to the power spectra of the first 15 years of the simulation, and the dashed lines of the last 15 years. It is clear that in both periods the magnetic power spectrum has its peak in the dipolar term with  $\ell = 1$ . Moreover, during the early times, 99.54% of the magnetic energy is in the dipole component. At late times, this reduces to 97.27% but this is not because the dipole amplitude decreases, but due to an increase in the contribution smaller scales ( $\ell > 1$ ). The time evolution of the dipole amplitude ( $\ell = 1$ ), normalized by its initial value, is shown in the right panel of Fig. 9. In the cases with Dipole B and B\*, the dipole amplitude is essentially constant over time, with the exception of DipBt\* where the dipole decays more than in the other runs. Nevertheless, the fraction of the energy in the dipole changes from 99.33% at early to 94.19% at late times. In the runs with Dipole A, the dipole decays faster than in the rest of simulations. For example, in DipAt the dipole contains 98.98% of the energy at early times, but at the late times this fraction drops to 50.28%. This suggests that while Dipole B can enhance the dynamo and possibly land it on the strong field branch, Dipole A is not conducive to this.

### 4.3. Differential rotation

The averaged rotation rate is given by

$$\bar{\Omega}(\varpi, z) = \Omega_0 + \bar{U}_\phi(\varpi, z)/\varpi, \quad (20)$$

where  $\varpi = r \sin \theta$  is the cylindrical radius. Additionally, the averaged meridional flow is

$$\bar{U}_{\text{mer}}(\varpi, z) = (\bar{U}_\varpi, 0, \bar{U}_z). \quad (21)$$

To quantify the amplitude of the radial and latitudinal differential rotation, we use the following parameters (Käpylä et al. 2013)

$$\Delta_\Omega^{(r)} = \frac{\bar{\Omega}(r_{\text{top}}, \theta_{\text{eq}}) - \bar{\Omega}(r_{\text{bot}}, \theta_{\text{eq}})}{\bar{\Omega}(r_{\text{top}}, \theta_{\text{eq}})}, \quad (22)$$

$$\Delta_\Omega^{(\bar{\theta})} = \frac{\bar{\Omega}(r_{\text{top}}, \theta_{\text{eq}}) - \bar{\Omega}(r_{\text{top}}, \bar{\theta})}{\bar{\Omega}(r_{\text{top}}, \theta_{\text{eq}})}, \quad (23)$$

where  $r_{\text{top}} = 0.9R$  and  $r_{\text{bot}} = 0.1R$  are the radius near the surface and center of the star, respectively.  $\theta_{\text{eq}}$  is the latitude at the equator, and  $\bar{\theta}$  is an average of  $\bar{\Omega}$  between latitudes  $-\theta$  and  $\theta$ . Following Paper I, we define  $\Delta_\Omega^{\text{CZ}(r)}$  and  $\Delta_\Omega^{\text{CZ}(\bar{\theta})}$ , which are the same as Eqs. (22) and (23), but with  $r_{\text{top}} = 0.2R$  and  $r_{\text{bot}} = 0.05R$ , to study the differential rotation of the core.

The differential rotation parameters and the maximum meridional speeds are summarized in Table 3. From columns 3 and 4 of Table 3 we can conclude that MHDr2 already has an almost-rigid rotation close to the surface (see Fig. 8 of Paper I). However, the inclusion of a fossil field, irrespective of its type, reduces the amplitude of the differential rotation almost completely with typical values being  $|\Delta_\Omega^{(\bar{\theta})}(60^\circ)| \sim 10^{-6}$  and  $|\Delta_\Omega^{(\bar{\theta})}(75^\circ)| \sim 10^{-5}$ . A similar phenomena is observed in the convective core. In runs with Dipole A, the parameters of the core

maintain their sign ( $\Delta_\Omega^{\text{CZ}(r)}, \Delta_\Omega^{\text{CZ}(\bar{\theta})} > 0$ ), which indicates that the solar-like state from run MHDr2 is still present in these simulations. However, the amplitude of this coefficient is significantly weaker than in MHDr2. In the cases with Dipole B\* and Dipole B, the differential rotation of the core is significantly quenched as well. The solar-like state changes to an anti-solar regime, where  $\Delta_\Omega^{\text{CZ}(r)}, \Delta_\Omega^{\text{CZ}(\bar{\theta})} < 0$  typically. This is also visible in the first three panels of figure 10. The initially retrograde column close to the axis of rotation (see panel(d) of Fig.1) of MHDr2 changes to a weak prograde column in the runs with the enhanced dynamo. A similar transition was reported by Augustson et al. (2016), between their hydrodynamic and the superequipartition magnetic cases (see their Fig. 7). Moreover, our results are also in agreement with Featherstone et al. (2009), as the prominent retrograde columns of the progenitor cases are replaced by regions of slightly prograde rotation, and the radiative envelope is almost rotating solidly after the inclusion of a poloidal fossil field. Finally, from the last column of Table 3 we conclude that there is no apparent relation between  $\beta$  and the maximum meridional speeds.

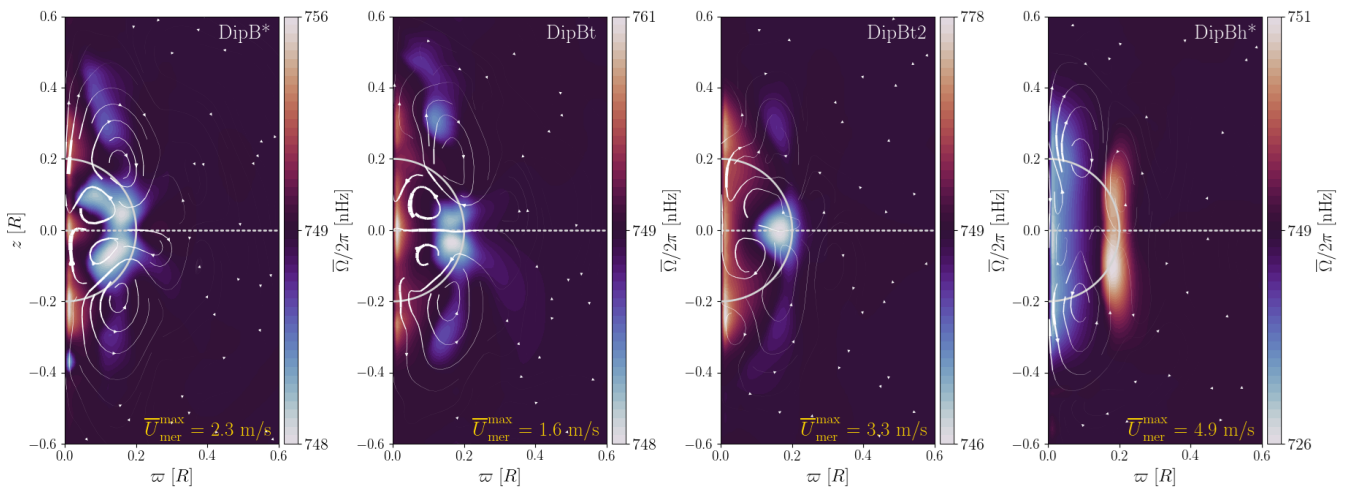
## 5. Discussion and conclusions

The interaction between the core dynamo of a  $2.2M_\odot$  A-type star and an imposed purely poloidal fossil field with a dipolar surface strength of 6 kG, was explored using star-in-a-box MHD simulations (Dobler et al. 2006; Käpylä 2021). Furthermore, as the observed magnetic fields from Ap stars are typically misaligned with their rotational axis, obliquity angles  $\beta$  between  $0^\circ$  and  $90^\circ$  were tested. In the case where none of the magnetic field lines are closed inside the star (Dipole A), the core dynamo does not seem to be affected, irrespective of the chosen inclination. A possible explanation for this, is that the internal poloidal field of this initial condition ( $B_{\text{rms}}^{\text{pol}} \approx 6$  kG) is too weak to excite a new dynamo mode. Moss (2004) found that an external poloidal magnetic field of the same order as the poloidal component of the initial dynamo, is sufficient to significantly affect the dynamo. Following this argument, the dipole field needs to be at least  $B_{\text{rms}}^{\text{pol}} = 19$  kG (see Table 1) to change the nature of the core dynamo in our simulations, which is not achieved by Dipole A. In the cases where the magnetic field lines of the imposed field are closed inside the star (Dipole B and Dipole B\*),  $B_{\text{rms}}^{\text{pol}}$  in this region is strong enough to affect the core dynamo. More specifically, with most of the explored inclinations (from  $0^\circ$  to  $85^\circ$ ) the initially cyclic and hemispheric solutions of the dynamo changes to a mainly dipolar quasi-stationary configuration. The strength of this enhanced dynamo seems to be inversely related to the obliquity angle of the fossil field. The horizontal cases ( $\beta = 90^\circ$ ) start with the mentioned quasi-stationary solution, but decay after a few years, proving to be unstable. Inertia seems to play an important role in the geometry of the dynamo, that is, dipolar solutions are expected when inertia is weak (Christensen & Aubert 2006; Soderlund et al. 2012). This is in agreement with the enhanced dynamos, where  $\text{Co} \approx 15$ , and where the addition of a strong dipole moves the simulation permanently to the dipolar branch. When inertia becomes relevant, multipolar solutions are expected. The runs in the dipolar branch by Gastine et al. (2012) have similar dipole-dominated solutions like the enhanced cases, while runs in the multipolar branch have a cyclic and hemispheric nature like MHDr2 (see their Fig.12). The enhanced dynamos have superequipartition fields, such that  $E_{\text{mag}}/E_{\text{kin}}$  is between 1.1 and 5.2. These ratios are comparable to the fast rotators of Augustson et al. (2016). However, we do

**Table 3.** Differential rotation parameters and the maximum meridional flow  $\bar{U}_{\text{mer}}^{\text{max}}$  from all the simulations.

Run	$\Delta_{\Omega}^{(r)}$	$\Delta_{\Omega}^{(\bar{\theta})}(60^{\circ})$	$\Delta_{\Omega}^{(\bar{\theta})}(75^{\circ})$	$\Delta_{\Omega}^{\text{CZ}(r)}$	$\Delta_{\Omega}^{\text{CZ}(\bar{\theta})}(60^{\circ})$	$\Delta_{\Omega}^{\text{CZ}(\bar{\theta})}(75^{\circ})$	$\bar{U}_{\text{mer}}^{\text{max}}$ [m/s]
MHDr2	0.0544	$-3.683 \cdot 10^{-5}$	$-5.329 \cdot 10^{-4}$	0.1201	0.0714	0.1196	6.0
DipA	0.0040	$3.179 \cdot 10^{-6}$	$4.803 \cdot 10^{-5}$	0.0075	0.0370	0.0622	3.6
DipAt	0.0039	$-3.771 \cdot 10^{-6}$	$1.163 \cdot 10^{-5}$	0.0084	0.0221	0.0375	2.1
DipAt2	0.0051	$7.799 \cdot 10^{-6}$	$3.331 \cdot 10^{-5}$	0.0106	0.0577	0.0962	1.8
DipAh	0.0041	$3.576 \cdot 10^{-6}$	$-1.330 \cdot 10^{-5}$	0.0087	0.0207	0.0331	5.2
DipB*	-0.0010	$8.513 \cdot 10^{-6}$	$2.724 \cdot 10^{-5}$	-0.0022	0.0013	-0.0014	2.4
DipBt*	-0.0000	$2.671 \cdot 10^{-6}$	$-5.959 \cdot 10^{-6}$	-0.0005	0.0016	0.0009	1.6
DipBt2*	-0.0010	$-1.700 \cdot 10^{-7}$	$4.614 \cdot 10^{-6}$	-0.0027	-0.0065	-0.0108	3.3
DipBt3*	-0.0005	$5.966 \cdot 10^{-6}$	$3.665 \cdot 10^{-5}$	-0.0014	-0.0134	-0.0228	1.9
DipBt4*	0.0010	$1.362 \cdot 10^{-5}$	$2.461 \cdot 10^{-5}$	-0.0017	-0.0056	-0.0136	1.6
DipBh*	0.0028	$1.096 \cdot 10^{-7}$	$-3.867 \cdot 10^{-6}$	0.0064	0.0162	0.0326	2.1
DipB	-0.0010	$3.081 \cdot 10^{-6}$	$1.748 \cdot 10^{-5}$	-0.0025	-0.0027	-0.0049	2.3
DipBt	-0.0013	$6.263 \cdot 10^{-6}$	$1.567 \cdot 10^{-5}$	-0.0031	-0.0021	-0.0029	2.2
DipBt2	0.0001	$7.119 \cdot 10^{-6}$	$-1.860 \cdot 10^{-7}$	-0.0048	-0.0048	-0.0085	2.0
DipBh	0.0118	$1.527 \cdot 10^{-5}$	$4.781 \cdot 10^{-7}$	0.0252	0.0407	0.0730	4.9

**Notes.** The differential rotation parameters follow Eqs. (22) and (23).



**Fig. 10.** Profiles of the temporally and azimuthally averaged rotation rate  $\bar{\Omega}(\varpi, z)$  of selected runs with Dipole B and Dipole B\* (clipped at  $r = 0.6R$ ). The streamlines indicate the mass flux due to meridional circulation. The maximum meridional flow speed is indicated in the lower right side of the plot. The dashed line at  $z = 0$  represents the equator.

not find fields as strong as those reported by Featherstone et al. (2009) in their mixed-case ( $E_{\text{mag}}/E_{\text{kin}} \approx 10$ ).

In the enhanced cases, the strong magnetic fields affect the profiles of differential rotation, inducing nearly rigid rotation in the radiative envelope, and changing the differential rotation in the convective core from solar-like to anti-solar. A similar transition was reported by Featherstone et al. (2009) after the inclusion of a fossil field, and by Augustson et al. (2016) between the hydrodynamic and magnetic cases. It is worth mentioning that in the enhanced cases, the amplitude of the anti-solar differential rotation is much weaker than that from the original solar-like state. This is a direct consequence of magnetic quenching of differential rotation (see, e.g. Brun et al. 2004, 2005; Käpylä et al. 2017; Bice & Toomre 2023). Our results are in agreement with asteroseismic observations, as most of the observed intermediate-mass MS stars ( $M < 9M_{\odot}$ ) have nearly solid body rotation (Kurtz et al. 2014; Bowman 2021, 2023).

Toroidal fields roughly comparable to the poloidal ones are found in the statistically steady state at the stellar surface. These

fields are most likely transported from the enhanced core dynamo to the surface due to turbulent diffusion. In the relaxed state the surface magnetic field does not show signs of decay. In runs with Dipole B and B\* the surface field has typically between 94% and 97% of its energy concentrated in the dipole mode. The relation between  $\beta$  and  $B_{\text{rms}}$  is not prominent on the stellar surface (see right panels of Fig. 8), therefore,  $\beta$  seems to affect mostly the enhanced core dynamo. In Ap/Bp stars, the distribution of  $\beta$  seems to be random, with no apparent relation to the surface dipolar strength (Hubrig et al. 2007; Sikora et al. 2019; Shultz et al. 2019). The surface poloidal field strength is significantly weaker in simulations with horizontal dipoles than in the rest of cases.

In this work we explored very idealized tilted poloidal fields, roughly based on the observed large-scale magnetic fields of Ap/Bp stars. However, fossil fields coming from the PMS evolution could in principle have complex topologies. Unfortunately, very little is known about the geometries and topologies of Herbig Ae/Be stars, the progenitors of Ap/Bp stars (see e.g. Hubrig

et al. 2020). A logical next step would be to model the PMS evolution of these stars, similarly to [Emeriau-Viard & Brun \(2017\)](#), but concentrating on Herbig Ae/Be stars. The obtained magnetic fields will probably become mainly dipolar during the main-sequence, as shown by [Braithwaite & Nordlund \(2006\)](#), even random initial magnetic fields can potentially relax into a dipolar structure. However, how a more realistic configuration can affect an existing core dynamo remains unknown.

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